ON THE NATURE OF THE "PINCH EFFECT" AND SOME OTHER PROBLEMS IN THE THEORY OF FAILURE

G. P. Cherepanov

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P. W. Bridgman's book [1] describes several forms of failure characteristic of high pressures. The explanation of these phenomena proposed below is based on modern concepts concerning the rupture of brittle and plastic solids.

§1. The pinch effect consists in the following. A continuous cylinder is subjected to a pressure acting on the outside cylindrical surface only; the ends are not under pressure. When the pressure attains a value roughly equal to the ultimate strength in pure tension, parts of the cylinder fail, usually somewhere near the middle, but very seldom in the zone adjoining the seals. The impression is created that the cylinder has failed under a tensile load applied directly to its projecting ends.... If the cylinder is made of a brittle material such as, for instance, glass or steel having the hardness of glass failure produces a clearly expressed plane perpendicular to the axis, but, on the other hand, if the cylinder is made of a material that remains capable of yielding up to failure, mild steel for instance, there is marked necking at the point of fracture. On the whole, the fracture is very similar in form to that obtained in an ordinary tensile test ([1], page 96). Bridgman's explanation of the pinch effect, based on the criterion of maximum elongation, is unsatisfactory.

The principal elastic stresses in the cylinder are obviously equal to:

$$\sigma_r = -p, \qquad \sigma_\theta = -p, \qquad \sigma_z = 0\,. \tag{1}$$

Here r, θ and z are cylindrical coordinates (the z axis coincides with the cylinder axis), and p is the lateral pressure.

A cylinder of ideally plastic material will obviously fail at a pressure p equal to σ_s, where σ_s is the yield point, because precisely at this pressure it goes over into the plastic state ($2 \max \tau = |\sigma_r - \sigma_z| = \sigma_s$). At failure, these values of pressure and slip-surface direction coincide with the corresponding values for the case of a rod subjected to a tensile stress $\sigma_z = \sigma_s$.

Let us now consider the cylinder material ideally brittle or quasi-brittle [2]. A characteristic property of such materials is the presence of a large number of surface microcracks or defects. According to the brittle fracture theory [2], the tensile strength of a cylinder is

$$\sigma = \frac{K}{\sqrt{R}}\, f(\alpha_1, \ldots, \alpha_n) \tag{2}$$

where K is the cohesion coefficient, and R is the cylinder radius. The function $f(\alpha_1, \ldots, \alpha_n)$ depends on the dimensionless form factors $\alpha_1, \ldots, \alpha_n$; it is fully defined by the geometry of the surface microcracks. Hence, the largest cracks normal to the surface play the greatest role. In Bridgman's experiment the pressure was transmitted to the lateral face of the cylinder by either a gas or a liquid. It is quite obvious that gas or liquid penetrated into the surface microcracks and produced a pressure on their walls equal to the pressure on the lateral face of the cylinder. On solving the corresponding problem in crack theory, it is easy to see that, at least where the microcracks are all normal to the surface, the pressure at failure will coincide with the value of σ given by Eq. (2). Strictly speaking, the function $f(\alpha_1, \ldots, \alpha_n)$ is different for different cylinders of the same length and radius. However, in the case of cylinders of the same material and similar fabrication, with the same past history and at the same experimental temperature, it can safely be assumed that these differences are immaterial. Hence we get Bridgman's result concerning the approximate equality of the limiting pressure and the ultimate tensile strength.

Bridgman also describes another case of failure where the elongation does not determine rupture and the individual stresses and strains are all compressive. A finite steel cylinder is tightly fitted into an ebonite tube of the same length. Both are subjected to hydrostatic pressure acting over the entire outside surface. Failure is somewhat as if a cone had been pushed into the tube, stretching it to breaking point. The explanation is analogous to that offered for the "pinch effect" and is based on the fact that the stress σ_θ in the tube is always less in absolute value than the outside pressure. It is easy to find the limiting extremal pressure p using our previous assumptions:

$$p^* = \frac{E_1 \sigma_p}{(1 - 2\nu_2) E_1 - (1 - 2\nu_1) E_2} \tag{3}$$

where σ_p is the tensile strength of the tube material (ebonite); ν is Poisson's ratio; E is Young's modulus; the subscript 1 refers to the material of the continuous cylinder (steel); and the subscript 2 refers to the tube materal (ebonite).

§2. Let us examine the question of compressive strength in relation to brittle or quasi-brittle materials. This problem is also of interest in connection with Bridgman's experiments pertaining to the failure of thick-walled brittle cylinders [1].

Let a continuous brittle cylinder with a free lateral face be subjected to axial compression. There appears to be no reason for failure, because plastic or viscous flow is excluded. However, the presence of large numbers of microcracks and defects in real brittle solids, especially close to the surface, causes the solid to be split up, as it were, by the defects into a number of rods of very complex shape, as a result of which it loses its elastic stability and fails. Hence, the critical pressure is completely determined by the size and distribution of the defects. Accurate calculation is impossible in this case, but approximate formulas can be derived on the basis of dimensional analysis and certain reasonable assumptions.

Let the characteristic dimension of the cracks in the brittle solid be denoted by l and let the characteristic dimension of the end region of the crack, where the cohesive forces act, be denoted by d. Let the intensity of the cohesive forces in the end region be of the order of G. Then the compressive strength of a brittle solid can obviously be written in the form

$$\sigma_- = GF\,(l\,/\,d)\,. \tag{4}$$

Note that formula (4) does not take into account the effect of the size of the specimen itself; it is obvious that this effect is negligibly small only when the characteristic dimension of the crack is small in comparison with the characteristic linear dimension of the solid.

Following the example of G. I. Barenblatt [2], we shall make the natural assumption that

$$d \ll l\,. \tag{5}$$

Hence, formula (4) can be written in the form

$$\sigma_- \sim G \tag{6}$$

or, keeping in mind that the cohesion coefficient K is of the order of $Gd^{1/2}$ and the tensile strength of σ_+ of the order of $Kl^{-1/2}$, in the form

$$\sigma_-/\sigma_+ \sim \sqrt{l/d} \tag{7}$$

Obviously, for an ideally brittle solid the intensity of the cohesive forces is of the order of Young's modulus E and for a plastic or near-plastic solid of the order of the yield point. Therefore, in accordance with formula (6), the proximity of the compressive strength to the modulus of elasticity E can serve as a qualitative characteristic of the proximity of the corresponding material to a perfectly brittle one. Formula (7) can be used to verify the realizability of the smallness hypothesis (5) in the case of the natural microcracks always present in a real solid, on the basis of macroscopic tests.

By way of comparison, we give data for the mechanical properties of some quasi-brittle materials; the figures (in kg/mm^2) were taken from handbook [3]:

silicate glasses

$$\sigma_+ = 3 - 9, \qquad \sigma_- = 50 - 200, \qquad E = (5 - 8.5)\cdot 10^3\,,$$

cast stone

$$\sigma_+ = 2, \qquad \sigma_- = 20, \qquad E = 11.000,$$

acid-resistant ceramics

$$\sigma_+ = 1.15 - 11, \qquad \sigma_- = 35 - 160, \qquad E = 42 - 70\,,$$

porcelain

$$\sigma_+ = 2.5 - 3.5, \qquad \sigma_- = 45 - 55, \qquad E = 60 - 80\,.$$

The low compressive strength σ_- for glasses and viscoelastic solids, compared with the modulus of elasticity, is due to the intense stress relaxation near the ends of the cracks.

REFERENCES

1. P. Bridgman, The Physics of High Pressure, London, 1931.
2. G. I. Barenblatt, "Mathematical theory of balanced cracks formed in brittle failure," PMTF, no. 4, 1961.
3. Handbook of Engineering Materials [in Russian], vol. 4, Gostekhizdat, Moscow, 1960.

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